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The one particle quantum mechanics is considered in the frame of a N-body classical kinetics in the phase space. Within this framework, the scenario of a subquantum structure for the quantum particle, emerges naturally, providing an ontological support to the orthodox quantum mechanics. This approach to quantum mechanics, constitutes a deductive and direct method which, in a self-consistent scheme of a classical kinetics, allows us: i) to obtain the probabilistic nature of the quantum description and to interpret the wave function ψ according to the Copenhagen school; ii) to derive the quantum potential and then the Schrödinger equation; iii) to calculate the values of the physical observables as mean values of the associated quantum operators; iv) to obtain the Heisenberg uncertainty principle.

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I. INTRODUCTION

The problem still open of interpretation and derivation of quantum mechanics in the frame of classical physics was first arisen in the same year in which the Schrödinger equation first appeared. Essentially two different approaches to this problem exist.

In the earliest, the hydrodynamic approach, introduced in 1926, Madelung showed that the ansatz $\psi = \rho^{1/2}\exp(i\mathrm{S}/\hbar)$ transforms the time dependent Schrödinger equation into two hydrodynamic equations [1]. The first one is the evolution equation for ρ and is a continuity equation while the second is the evolution equation for S and can be viewed as an Euler equation or as an Hamilton-Jacobi equation with an extra contribution to the energy corresponding to the presence of the quantum potential. The quantum fluid obtained in this way starting from the Schrödinger equation was studied extensively by various authors [2–16]. Takabayashi has shown that the quantum potential leads to a particular form for the stress tensor of the quantum fluid [5]. In the hydrodynamic approach which can be viewed essentially as an indirect method that permits to interpret the quantum mechanics, the particle takes the form of a highly localized inhomogeneity that moves with the local fluid velocity so that the quantum fluid appears as the medium which transport the particle.

The second, the stochastic mechanics approach [17–30], is more ambitious being a direct method which wants to derive the Schrödinger equation in the frame of a particular classical dynamics and gives an explicit representation of quantum mechanics in term of classical probability densities for a particle undergoing a brownian motion with a diffusion constant proportional to Planck's constant and inversely proportional to particle mass. This quantum brownian motion is nondissipative in contrast to the standard one and is responsible for the disordering of the particle trajectory and does not take energy from it. The dynamics of this quantum brownian particle is described in the frame of the Einstein-Smoluchowski theory which is an approximation of the Ornestein-Uhlenbeck theory governing the standard brownian motion. The main critics to the stochastic approach is that the quantum brownian motion cannot be obtained from first principles and appears built on specifically in order to produce the quantum mechanics. In this way it seems that the stochastic approach moves the problem to another level [31–35].

The idea that the quantum state corresponds to a classical statistical ensemble is common in both the approaches above described. This idea, so commonly accepted, we believe must to be at the base of any attempt to derive and interpret the quantum mechanics. Let's pose now the question: does it exist a rigorous way to describe a time depending classical statistical ensemble starting from some simple principle without making use of any extra assumption in order to obtain the quantum mechanics? In other words, is it possible to build a program in which the microscopic motion, underlying the quantum mechanics, is described by a rigorous dynamics different from the brownian one? The answer to this question is affirmative.

In the present paper we will show that the quantum state corresponds exactly to a subquantum statistical ensemble whose time evolution is governed by a classical kinetics in the phase space. The idea that the classical kinetics could be underlying quantum mechanics is new and is proposed here the first time.

II. CLASSICAL KINETICS

Let us consider a set of N identical interacting particles in a n-dimensional physical space ($n = 1, 2, 3$) which in the following we call *monads*. Each monad has a mass μ and obeys to the laws of classical physics. We suppose

that this set classical particles constitutes a statistical ensemble described in the $2n$-dimensional phase space by the distribution function $f(t, \boldsymbol{x}, \boldsymbol{v})$ with

$$\int f\, d^n v d^n x = N \quad , \tag{1}$$

and postulate that the dynamics of this set of particles is governed by the following kinetic equation:

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\frac{\partial f}{\partial \boldsymbol{x}} + \frac{\boldsymbol{F}}{\mu}\frac{\partial f}{\partial \boldsymbol{v}} = C(f) \quad . \tag{2}$$

The external force $\boldsymbol{F}$, acting on the generic monad, is supposed to derive from a scalar potential, $\boldsymbol{F} = -\partial\mathcal{V}/\partial\boldsymbol{x}$. However the theory developed here can be easily generalized in order to treat particle systems immersed in an electro-magnetic field [36]. We don't make any assumption about the nature of the interaction between the monads except that, during the collisions the monad number, momentum and energy are conserved. This assumption implies that the three functions $g_1(\boldsymbol{v}) = 1$, $g_2(\boldsymbol{v}) = \boldsymbol{v}$ and $g_3(\boldsymbol{v}) = \boldsymbol{v}^2$ are the collisional invariants of the system and thus the collisional integral $C(f)$ satisfies the conditions [37,38]:

$$\int C(f) d^n v = \int \boldsymbol{v} C(f) d^n v = \int \boldsymbol{v}^2 C(f) d^n v = 0 \quad . \tag{3}$$

In the following, we indicate with $\mathcal{G}(t, \boldsymbol{x}, \boldsymbol{v})$ the density of a given physical quantity so that $\mathcal{G} f d^n v d^n x$ represents its value around the point $(\boldsymbol{x}, \boldsymbol{v})$ while its total value is $\int \mathcal{G} f d^n v d^n x$. We consider now the projection of the system dynamics in the n-dimensional physical space, where the distribution function is $\rho(t, \boldsymbol{x}) = \int f d^n v$. The mean value of $\mathcal{G}(t, \boldsymbol{x}, \boldsymbol{v})$ in the point $\boldsymbol{x}$ is given by:

$$< \mathcal{G}(t, \boldsymbol{x}, \boldsymbol{v}) >_v = \frac{\int \mathcal{G}(t, \boldsymbol{x}, \boldsymbol{v}) f(t, \boldsymbol{x}, \boldsymbol{v}) d^n v}{\int f(t, \boldsymbol{x}, \boldsymbol{v}) d^n v} \quad , \tag{4}$$

and represents the density of the physical quantity in the physical space. Consequently, its total value can be written as $\int \rho < \mathcal{G} >_v d^n x$. We define the densities in the physical space of some quantities used in following. The density of current is given by $\boldsymbol{u} = < \boldsymbol{v} >_v$. The density of stress tensor is given by

$$\sigma_{ij} = \mu < (v_i - u_i)(v_j - u_j) >_v \quad , \tag{5}$$

and is a symmetric tensor $\sigma_{ij} = \sigma_{ji}$ of rank two. The density of the heat flux vector is given by

$$h_i = \frac{1}{2}\mu < |\boldsymbol{v} - \boldsymbol{u}|^2 (v_i - u_i) >_v \quad . \tag{6}$$

The density of energy is given by

$$E = \frac{1}{2}\mu < \boldsymbol{v}^2 >_v + \mathcal{V} = \frac{1}{2}\mu \boldsymbol{u}^2 + \varepsilon + \mathcal{V} \quad , \tag{7}$$

where ε is the density of the internal energy defined as:

$$\varepsilon = \frac{1}{2}\mu \left(< \boldsymbol{v}^2 >_v - < \boldsymbol{v} >_v^2 \right) \quad . \tag{8}$$

It results that $\varepsilon = \sigma_{ii}/2$ and the pressure is defined as $\pi = \rho\, \sigma_{ii}/n$. Then we can write the equation of state for the system as $\pi = 2\, \rho\, \varepsilon / n$.

Multiplying Eq. (2) by the three collisional invariants and after integration with respect to v, the three following hydrodynamic equations can be obtained [37]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \boldsymbol{x}}(\rho \boldsymbol{u}) = 0 \quad , \tag{9}$$

$$\frac{\partial}{\partial t}(\mu \rho u_i) + \frac{\partial}{\partial x_j}(\rho \varphi_{ij}) + \rho \frac{\partial \mathcal{V}}{\partial x_i} = 0 \quad , \tag{10}$$

$$\frac{\partial}{\partial t}(\rho E) + \frac{\partial}{\partial \boldsymbol{x}}(\rho \boldsymbol{s}) - \rho \frac{\partial \mathcal{V}}{\partial t} = 0 \quad , \tag{11}$$

where $\varphi_{ij} = \mu u_i u_j + \sigma_{ij}$ is the momentum flux density tensor and $s_i = E u_i + \sigma_{ij} u_j + h_i$ is the energy flux density vector.

Eq. (9) is the continuity equation for the system which behaves in the physical space as a fluid. This equation implies the conservation of the particle number:

$$N = \int \rho\, d^n x \quad . \tag{12}$$

Eq. (10) is the Euler equation for the fluid and in absence of external forces $\partial\mathcal{V}/\partial\boldsymbol{x} = 0$, implies the conservation of the system total momentum:

$$\boldsymbol{P} = \int \mu\, \rho\, \boldsymbol{u}\, d^n x \quad . \tag{13}$$

Finally, Eq.(11) governs the evolution of energy and in the case of time independent forces $\partial\mathcal{V}/\partial t = 0$ implies the conservation of the system total energy:

$$H = \int E\, \rho\, d^n x \quad . \tag{14}$$

By taking into account Eq. (9) we rewrite Eq. (10) in the following form

$$\mu \frac{D\boldsymbol{u}}{Dt} = \boldsymbol{\mathcal{F}}^{(\sigma)} + \boldsymbol{F} \quad , \tag{15}$$

where $D/Dt = \partial/\partial t + \boldsymbol{u} \cdot \partial/\partial \boldsymbol{x}$ is the total time derivative while the hydrodynamic force $\boldsymbol{\mathcal{F}}^{(\sigma)}$, is originated from the stress tensor and takes the form

$$\mathcal{F}_j^{(\sigma)} = -\sigma_{jk}\frac{\partial \xi}{\partial x_k} - \frac{\partial \sigma_{jk}}{\partial x_k} \quad , \tag{16}$$

with

$$\xi = \ln \rho \quad . \tag{17}$$

It is interesting to note that the force $-\mu D\boldsymbol{u}/Dt$ assumes a Lorentz like form

$$-\mu\frac{D\boldsymbol{u}}{Dt}=\mu\boldsymbol{\mathcal{E}}^{(u)}+\mu\boldsymbol{u}\times\boldsymbol{\mathcal{B}}^{(u)}\quad , \tag{18}$$

where the fields $\boldsymbol{\mathcal{E}}^{(u)}$ and $\boldsymbol{\mathcal{B}}^{(u)}$ can be derived from the four vector potential

$$\mathcal{A}^{(u)}=\left(\mathcal{A}_0^{(u)},\boldsymbol{\mathcal{A}}^{(u)}\right)=\left(\frac{1}{2}\boldsymbol{u}^2,\boldsymbol{u}\right)\quad . \tag{19}$$

by means of

$$\boldsymbol{\mathcal{E}}^{(u)}=-\frac{\partial\mathcal{A}_0^{(u)}}{\partial\boldsymbol{x}}-\frac{\partial\boldsymbol{\mathcal{A}}^{(u)}}{\partial t}\quad , \tag{20}$$

$$\boldsymbol{\mathcal{B}}^{(u)}=\frac{\partial}{\partial\boldsymbol{x}}\times\boldsymbol{\mathcal{A}}^{(u)}\quad . \tag{21}$$

The structural similarity between $-\mu D\boldsymbol{u}/Dt$ and the Lorentz force it is important when we treat systems in the presence of external electromagnetic fields [36].

III. DERIVATION OF QUANTUM POTENTIAL

The absence of the magnetic term in the external force $\boldsymbol{F}$ permits us to assume that the particle system is spinless [5–7]. Then we can set $\boldsymbol{\mathcal{B}}^{(u)}=0$ and consequently we have that $\boldsymbol{u}$ is irrotational, $(\partial/\partial\boldsymbol{x})\times\boldsymbol{u}=0$, and can be derived from a scalar potential

$$\boldsymbol{u}=\frac{1}{\mu}\frac{\partial S}{\partial\boldsymbol{x}}\quad . \tag{22}$$

At this point we observe that all the terms in Eq.(15) are irrotational. In particular also the stress force $\boldsymbol{\mathcal{F}}^{(\sigma)}$ is irrotational, $(\partial/\partial\boldsymbol{x})\times\boldsymbol{\mathcal{F}}^{(\sigma)}=0$, and it can be derived from a scalar potential,

$$\boldsymbol{\mathcal{F}}^{(\sigma)}=-\frac{\partial\mathcal{W}}{\partial\boldsymbol{x}}\quad , \tag{23}$$

so that for spinless systems we have the following condition:

$$\sigma_{jk}\frac{\partial\xi}{\partial x_k}+\frac{\partial\sigma_{jk}}{\partial x_k}=\frac{\partial\mathcal{W}}{\partial x_j}\quad . \tag{24}$$

The vectorial equation (15) becomes now a scalar equation:

$$\frac{\partial S}{\partial t}+\frac{1}{2\mu}\left(\frac{\partial S}{\partial\boldsymbol{x}}\right)^2+\mathcal{W}+\mathcal{V}=0\quad . \tag{25}$$

We remark that Eq. (24) allows us to link the quantities σ_{jk} and $\mathcal{W}$. A first solution of Eq. (24), describing a classical Eulerian fluid, is given by $\sigma_{jk}=\frac{2}{n}\,\varepsilon(\xi)\delta_{jk}$ and $\mathcal{W}=\frac{2}{n}\,\varepsilon(\xi)+\frac{2}{n}\int\varepsilon(\xi)d\xi$ where the internal energy $\varepsilon(\xi)$ is an arbitrary algebraic function.

In the following, we will show that Eq.(24) admits a second more interesting and less evident solution. If this solution exists both terms in the left hand side of (24) will take the form $\partial(...)/\partial x_j$. In particular, for the second term we must pose $\sigma_{jk}=\partial a_k/\partial x_j$. Using the symmetry of the density of the stress tensor $\sigma_{jk}=\sigma_{kj}$ imposed by Eq.(5), we have that $a_k=\partial\alpha/\partial x_k$. Then the density of the stress tensor assumes the form

$$\sigma_{jk}=\frac{\partial^2\alpha}{\partial x_j\partial x_k}\quad , \tag{26}$$

with α an unknown scalar function depending on the field ξ. Eq. (24) can be written now in the form

$$\frac{\partial^2\alpha}{\partial x_j\partial x_k}\,\frac{\partial\xi}{\partial x_k}=\frac{\partial}{\partial x_j}\left(\mathcal{W}-\frac{\partial^2\alpha}{\partial x_k\partial x_k}\right)\quad . \tag{27}$$

By making the hypothesis that the function α is an algebraic function of the field ξ (it can be easily verified that this is the only possibility), the left hand side in (27), after developing the derivatives, becomes

$$\frac{\partial^2\alpha}{\partial x_j\partial x_k}\,\frac{\partial\xi}{\partial x_k}=\frac{\partial}{\partial x_j}\left[\frac{1}{2}\frac{d\alpha}{d\xi}\left(\frac{\partial\xi}{\partial x_k}\right)^2\right]+\frac{1}{2}\frac{d^2\alpha}{d\xi^2}\,\frac{\partial\xi}{\partial x_j}\left(\frac{\partial\xi}{\partial x_k}\right)^2$$

and must be written in the form $\partial(...)/\partial x_j$. This condition requires that

$$\frac{d^2\alpha}{d\xi^2}=0\quad . \tag{28}$$

After integration of this last equation we obtain $\alpha=c\xi+c_o$ with c and c_o arbitrary constants. The constant c_o, not influencing the value of σ_{jk}, can be set equal to zero. Consequently, we obtain

$$\sigma_{jk}=-\frac{\eta^2}{4\mu}\frac{\partial^2\xi}{\partial x_j\partial x_k}\quad , \tag{29}$$

$$\varepsilon=-\frac{\eta^2}{8\mu}\frac{\partial^2\xi}{\partial\boldsymbol{x}^2}\quad , \tag{30}$$

where we have posed $c=-\eta^2/4\mu$ in order to have $\varepsilon>0$. The integration constant $\eta\geq0$ of Eq. (28) is a real free parameter of the theory.

The potential $\mathcal{W}$ can be calculated immediately from (27) by posing $\alpha=-(\eta^2/4\mu)\xi$:

$$\mathcal{W}=-\frac{\eta^2}{4\mu}\left[\frac{1}{2}\left(\frac{\partial\xi}{\partial\boldsymbol{x}}\right)^2+\frac{\partial^2\xi}{\partial\boldsymbol{x}^2}\right]\quad . \tag{31}$$

Equations (29)and (31) describe a fluid different from the Eulerian one previously obtained. We call it quantum fluid, being $\mathcal{W}$ the Mandelug-Bohm quantum potential.

This fluid will be the object of our study in the following. We remark that the procedure used here to obtain the quantum potential evidences clearly its origin. In order to obtain (31) we have used the definition (16) of $\boldsymbol{\mathcal{F}}^{(\sigma)}$, the property $\sigma_{jk} = \sigma_{kj}$ enforced by the definition (5) of σ_{jk} and additionally assuming that $\boldsymbol{\mathcal{F}}^{(\sigma)}$ is a conservative force, being the system a spinless one. Besides, the fundamental constant $\hbar = N\eta$ emerges naturally as the integration constant of (28) and is a free parameter for the theory.

Finally we note that the expression of the density of stress tensor σ_{jk} given by Eq. (29), in the literature [5,15], appears in a different form, namely:

$$\sigma_{jk} = -\frac{\eta^2}{4\mu}\left(\frac{1}{\rho}\frac{\partial^2\rho}{\partial \boldsymbol{x}^2}\delta_{jk} - \frac{1}{\rho^2}\frac{\partial\rho}{\partial x_j}\frac{\partial\rho}{\partial x_k}\right).$$

Unfortunately, by considering the inverse procedure *Shrödinger equation → quantum fluid equations* adopted in the literature, both the expressions can be obtained, but no clear criterium exists to decide which of them is correct.

IV. SCHRÖDINGER EQUATION

The quantum fluid is described completely by the two scalar fields ρ, S whose evolution equations are (9) and (25), respectively. Alternatively, we can describe this fluid by means of the two scalar fields ξ, S whose evolution equations are:

$$\frac{\partial\xi}{\partial t} + \frac{1}{\mu}\frac{\partial^2 S}{\partial \boldsymbol{x}^2} + \frac{1}{\mu}\frac{\partial\xi}{\partial \boldsymbol{x}}\frac{\partial S}{\partial \boldsymbol{x}} = 0 \quad , \tag{32}$$

$$\frac{\partial S}{\partial t} - \frac{\eta^2}{4\mu}\frac{\partial^2\xi}{\partial \boldsymbol{x}^2} - \frac{\eta^2}{8\mu}\left(\frac{\partial\xi}{\partial \boldsymbol{x}}\right)^2 + \frac{1}{2\mu}\left(\frac{\partial S}{\partial \boldsymbol{x}}\right)^2 + \mathcal{V} = 0 \quad . \tag{33}$$

Given the structural resemblance of (32) and (33), we introduce the complex scalar field Ω through:

$$\Omega = \frac{\xi}{2} + \frac{i}{\eta}S \quad . \tag{34}$$

This field satisfies the condition $\int |\exp\Omega|^2 d^n x = N$ and is a many-valued function analogously to the function S, which is defined by means of (22). We can describe now the quantum fluid by using the complex field Ω which obeys the evolution equation:

$$i\eta\frac{\partial\Omega}{\partial t} = -\frac{\eta^2}{2\mu}\left[\frac{\partial^2\Omega}{\partial \boldsymbol{x}^2} + \left(\frac{\partial\Omega}{\partial \boldsymbol{x}}\right)^2\right] + \mathcal{V} \quad . \tag{35}$$

Eq. (35) is a Burger like equation which can be linearized by the Hopf-Cole transformation

$$\Omega = \mathrm{Log}\Psi \quad , \tag{36}$$

with Log representing the many-valued complex logarithmic function. If we introduce the total mass $m = N\mu$, the potential $V = N\mathcal{V}$ under which the entire system evolves and after setting $\hbar = N\eta$, Eq.(35) transforms to the Schrödinger equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial \boldsymbol{x}^2} + V\Psi \quad . \tag{37}$$

The field Ψ satisfies the normalization condition

$$\int |\Psi|^2 d^n x = N \quad , \tag{38}$$

and is a single-valued function. This last restriction implies that Ω is a many-valued logarithm so that S/η is a many-valued function, whose different values differ by integer multiples of 2π. This restriction on the values of S implies

$$\oint_\gamma \boldsymbol{u} d\boldsymbol{x} = j2\pi\hbar/m \quad , \tag{39}$$

and represents the quantization condition for the system [7,16].

The system of mass m, namely the quantum particle, can be described by means of the field Ψ taking into account that $|\Psi|^2 d^n x$ represents the number of monads around the point $\boldsymbol{x}$ or alternatively, in probabilistic terms, by means of the field $\psi = \Psi/\sqrt{N}$ according to the Copenhagen School. In the following we will continue to describe the system by using the field Ψ. The ansatz

$$\Psi = \rho^{1/2}\exp\left(\frac{i}{\hbar}\mathrm{S}\right) \quad , \tag{40}$$

with $\mathrm{S} = NS$, appears now as the transformation which linearizes the evolution equations (32), (33) and at the same time selects the quantized states of the system.

V. QUANTUM OPERATORS

We show now that the statistical nature of the quantum particle permits us to introduce the quantum operators whose mean values give the measured values of the physical observables. We recall that the measured value for a given physical observable of a statistical system is the mean value of its associated density $\mathcal{G}(t, \boldsymbol{x}, \boldsymbol{v})$ and can be calculated starting from the distribution function f:

$$< \mathcal{G}(t, \boldsymbol{x}, \boldsymbol{v}) >_{vx} = \frac{\int \mathcal{G}(t, \boldsymbol{x}, \boldsymbol{v}) f(t, \boldsymbol{x}, \boldsymbol{v}) d^n v d^n x}{\int f(t, \boldsymbol{x}, \boldsymbol{v}) d^n v d^n x} \quad . \tag{41}$$

In the case of the quantum fluid we can calculate the above mean value also starting from Ψ. In fact if we consider the identities

$$\int \boldsymbol{v}^l \, f \, d^n v \, d^n x = \int d^n x \, \Psi^* \left(\frac{-i\hbar}{m} \frac{\partial}{\partial \boldsymbol{x}} \right)^l \Psi \quad , \tag{42}$$

for $l = 1$ and $l = 2$ (see appendix) it is easy to verify that the physical observables momentum and kinetic energy can be obtained also as mean values of the two operators

$$\widehat{\boldsymbol{p}} = -i\hbar \frac{\partial}{\partial \boldsymbol{x}} \quad , \tag{43}$$

and

$$\widehat{T} = \frac{\widehat{\boldsymbol{p}}^{\,2}}{2m} \quad , \tag{44}$$

respectively, having defined the mean value of an operator as:

$$< \widehat{\mathcal{G}} >_x = \frac{\int \Psi^*(t, \boldsymbol{x}) \widehat{\mathcal{G}} \Psi(t, \boldsymbol{x}) d^n x}{\int \Psi^*(t, \boldsymbol{x}) \Psi(t, \boldsymbol{x}) d^n x} \quad . \tag{45}$$

When the density of the physical observable is an arbitrary function of the type $\mathcal{G}(t, \boldsymbol{x})$ it is immediate to verify that $\widehat{\mathcal{G}} = \mathcal{G}(t, \boldsymbol{x})$ and then we have

$$< \mathcal{G}(t, \boldsymbol{x}) >_{vx} = < \mathcal{G}(t, \boldsymbol{x}) >_x = < \widehat{\mathcal{G}} >_x \quad . \tag{46}$$

We remark at this point the statistical origin of the quantum operators. In fact we have introduced the quantum operators using the definition (41) of the mean value of the physical observables of a many body classical system and obviously the properties of the quantum fluid, namely the expressions of $\boldsymbol{u}$ and ε (see appendix).

In the following, we will write the total energy H of the quantum fluid in terms of the field Ψ. For this reason, we observe that any function $\mathcal{G}(t, \boldsymbol{x})$ can be written as

$$\mathcal{G} = \mathcal{G}^{(0)} + \mathcal{G}^{(I)} \quad , \tag{47}$$

where the first part has mean value equal to zero

$$< \mathcal{G}^{(0)} >_x = 0 \quad , \tag{48}$$

so that $< \mathcal{G} >_x = < \mathcal{G}^{(I)} >_x$. It is easy to verify that:

$$\varepsilon^{(I)} = \mathcal{W}^{(I)} = \frac{\eta^2}{8\mu} \left(\frac{\partial \xi}{\partial \boldsymbol{x}} \right)^2 \quad . \tag{49}$$

The total internal energy $\mathcal{H} = \int \varepsilon \rho \, d^n x$ of the system is given by

$$\mathcal{H} = \frac{\eta^2}{8\mu} \int \left(\frac{\partial \xi}{\partial \boldsymbol{x}} \right)^2 \rho \, d^n x \quad , \tag{50}$$

while the total energy H given by

$$H = \int \left(\frac{1}{2} \mu \boldsymbol{v}^2 + \mathcal{V} \right) f d^n v d^n x \quad . \tag{51}$$

assumes the form

$$H = \int \left[\frac{1}{2\mu} \left(\frac{\partial S}{\partial \boldsymbol{x}} \right)^2 + \frac{\eta^2}{8\mu} \left(\frac{\partial \xi}{\partial \boldsymbol{x}} \right)^2 + \mathcal{V} \right] \rho \, d^n x \quad , \tag{52}$$

and is the sum of two terms:

$$H = H_{cl} + \mathcal{H} \quad , \tag{53}$$

with

$$H_{cl} = \int \left[\frac{1}{2\mu} \left(\frac{\partial S}{\partial \boldsymbol{x}} \right)^2 + \mathcal{V} \right] \rho \; d^n x \quad . \tag{54}$$

The classical term H_{cl} of the total energy corresponds to a system with $\sigma_{ij} = 0$ while the quantum term $\mathcal{H}$ is originated by the internal structure of the system and is simply its *internal energy.*
The total energy can be written in the form

$$H = \int \left(\frac{\eta^2}{2\mu} \left| \frac{\partial \Omega}{\partial \boldsymbol{x}} \right|^2 + \mathcal{V} \right) \rho \; d^n x \quad . \tag{55}$$

while in terms of Ψ becomes

$$H = \int \left(\frac{\eta^2}{2\mu} \left| \frac{\partial \Psi}{\partial \boldsymbol{x}} \right|^2 + \mathcal{V} \left| \Psi \right|^2 \right) d^n x \quad . \tag{56}$$

When the system is described by means of the field Ψ is canonic and the evolution equation (37) can be obtained starting from the Hamiltonian (56) by using a variational principle:

$$i\eta \frac{\partial \Psi}{\partial t} = \frac{\delta H}{\delta \Psi^*} \quad . \tag{57}$$

Thus the ansatz $\Psi = \rho^{1/2} \exp(iS/\eta)$ can be viewed as a transformation which linearizes the evolution equation and at the same time conserves the canonicity of the system.

VI. UNCERTAINTY PRINCIPLE

The internal structure of the quantum particle implies a spatial dispersion and consequently an indetermination

$$(\Delta x)^2 = < (\boldsymbol{x} - < \boldsymbol{x} >_x)^2 >_x \quad , \tag{58}$$

on the measure of his position. In order to relate Δx with $\mathcal{H}$ we consider the inequality

$$\int \left| (< \boldsymbol{x} >_x - \boldsymbol{x}) \frac{\partial \rho}{\partial \boldsymbol{x}} \right| d^n x \geq N \quad , \tag{59}$$

which can be immediately verified if we observe that it becomes an obvious equality when the integrand is considered without the absolute value. This inequality can be written also as:

$$\int \sqrt{\frac{1}{N}\left(\boldsymbol{x}- <\boldsymbol{x}>_x\right)^2 \rho}\ \sqrt{\frac{1}{N}\left(\frac{\partial \xi}{\partial \boldsymbol{x}}\right)^2 \rho}\ d^n x \geq 1 \quad , \quad (60)$$

which, by taking into account the Schwartz inequality assumes the form

$$\frac{1}{N}\int \left(\boldsymbol{x}- <\boldsymbol{x}>_x\right)^2 \rho\ d^n x\ \frac{1}{N}\int \left(\frac{\partial \xi}{\partial \boldsymbol{x}}\right)^2 \rho\ d^n x \geq 1\ . \quad (61)$$

After remembering the definitions of Δx and $\mathcal{H}$ the inequality (61), transforms as

$$\Delta x \sqrt{2m\mathcal{H}} \geq \frac{\hbar}{2} \quad . \quad (62)$$

We show now that $\mathcal{H}$ is related also with the uncertainty on the measure of the momentum of the system

$$\begin{aligned}(\Delta p)^2 &= m^2 <(\boldsymbol{v}- <\boldsymbol{v}>_{vx})^2>_{vx} \\ &= <(\widehat{\boldsymbol{p}}- <\widehat{\boldsymbol{p}}>_x)^2>_x \quad . \end{aligned} \quad (63)$$

In fact by taking into account that $\mathcal{H}$ can be written also in the form

$$\mathcal{H} = \frac{1}{2} m <(\boldsymbol{v}- <\boldsymbol{v}>_v)^2>_{vx} \quad , \quad (64)$$

and after some simple algebra we obtain the relation:

$$(\Delta p)^2 = (\Delta p_{cl})^2 + 2m\mathcal{H} \quad , \quad (65)$$

where

$$(\Delta p_{cl})^2 = m^2 <(\boldsymbol{u}- <\boldsymbol{u}>_x)^2>_x \quad . \quad (66)$$

Then from (65) we conclude

$$\Delta p \geq \sqrt{2m\mathcal{H}} \quad . \quad (67)$$

This last inequality if combined with (62) leads immediately to the Heisenberg uncertainty principle:

$$\Delta x\ \Delta p \geq \frac{\hbar}{2} \quad . \quad (68)$$

The procedure used to obtain this principle shows clearly its statistical origin.

VII. CONCLUDING REMARKS

The theory here developed can be viewed as an approach constituting a deductive and direct method which, in a self-consistent scheme of a classical many-body physics, permits us i) to obtain the probabilistic nature of the quantum description and to interpret the wave function ψ according to the Copenhagen school; ii) to derive naturally the quantum potential and then the Schrödinger equation; iii) to calculate the values of the physical observables as mean values of certain associated operators, namely the quantum operators; iv) to obtain the Heisenberg uncertainty principle. Finally the fundamental constant $\hbar$ emerges naturally as an integration constant and represents a free parameter for the theory.

The theory can be viewed also as describing a mechanism allowing us to build the quantum particle starting from its constituents. The quantum particle of mass m turns out to be a statistical system having a spatial extension and an internal structure. It is composed by N identical subquantum interacting particles of mass μ, the monads. These monads obey the laws of classical physics and their dynamics is described in the phase space by the standard kinetic equation. We don't make any assumption about the nature of the interaction between the monads. We assume only that during collisions the monad number, momentum and energy are conserved. These assumptions, in the case of a spinless system, imply that its dynamics in the physical space is governed by the Schrödinger equation.

It is now clear that quantum mechanics is a non local, hidden variables theory, as suspected by some of its founders. In fact Eq. (2) describes a non relativistic subquantum statistical ensemble. The hidden dynamics, in which Einstein believed, seems to be the one imposed by the subquantum *monadic kinetics*. The orthodox quantum mechanics is an axiomatic theory and then, by taking into account the Gödel theorem, it can be obtained only in the framework of a wider metaquantum theory. This metaquantum theory appears to be now simply the subquantum monadic kinetics.

We discuss now briefly the problem concerning the locality in quantum physics, that were left unresolved in the twenty three year long debate between Einstein and Bohr and were reconsidered by Bell in 1964. It is well known that the Bell's inequality has been obtained in the framework of local, hidden variables and deterministic theories. This inequality is in disagreement both with quantum mechanics and experimental evidence. The reason of this disagreement now appears clear. Here we have obtained the quantum mechanics staring from the underlying monadic kinetics which is a non local, hidden variables and probabilistic theory. At this point, spontaneously arises the question, if it is possible to include the locality in quantum physics. After noting that a relativistic kinetics is a local and probabilistic theory, we can make the conjecture that *a subquantum relativistic monadic kinetics could be underlying a local quantum theory.*

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APPENDIX A

In this appendix we demonstrate the identity (42) in the case $l = 2$. We take into account the definition of the internal energy (8) and the expression of u_i and ε given by (22) and (30) respectively, we have:

$$\begin{aligned}\int v_i^2 f d^n x d^n v &= \int d^n x \rho < v_i^2 >_v = \int d^n x \rho (u_i^2 + 2\mu\varepsilon) \\ &= \int \rho \left[\left(\frac{1}{\mu} \frac{\partial S}{\partial x_i} \right)^2 - \frac{\eta^2}{4\mu^2} \frac{\partial^2 \ln \rho}{\partial x_i^2} \right] d^n x \\ &= -\frac{\eta^2}{\mu^2} \int \rho \Big[\frac{\partial}{\partial x_i} \left(\frac{1}{2\rho} \frac{\partial \rho}{\partial x_i} + \frac{i}{\eta} \frac{\partial S}{\partial x_i} \right) \\ &+ \left(\frac{1}{2\rho} \frac{\partial \rho}{\partial x_i} + \frac{i}{\eta} \frac{\partial S}{\partial x_i} \right)^2 \Big] d^n x \\ &= -\frac{\eta^2}{\mu^2} \int \Psi^* \Big[\Psi \frac{\partial}{\partial x_i} \left(\frac{1}{2\rho} \frac{\partial \rho}{\partial x_i} + \frac{i}{\eta} \frac{\partial S}{\partial x_i} \right) \\ &+ \left(\frac{1}{2\rho} \frac{\partial \rho}{\partial x_i} + \frac{i}{\eta} \frac{\partial S}{\partial x_i} \right)^2 \Psi \Big] d^n x \quad .\end{aligned}$$

From the ansatz $\Psi = \rho^{1/2} \exp(iS/\eta)$ we have

$$\frac{\partial \Psi}{\partial x_i} = \left(\frac{1}{2\rho} \frac{\partial \rho}{\partial x_i} + \frac{i}{\eta} \frac{\partial S}{\partial x_i} \right) \Psi \quad ,$$

so that Eq. (42) immediately is obtained

$$\begin{aligned}&\int v_i^2 f d^n x d^n v \\ &= -\frac{\eta^2}{\mu^2} \int \Psi^* \Big[\Psi \frac{\partial}{\partial x_i} \left(\frac{1}{2\rho} \frac{\partial \rho}{\partial x_i} + \frac{i}{\eta} \frac{\partial S}{\partial x_i} \right) \\ &+ \left(\frac{1}{2\rho} \frac{\partial \rho}{\partial x_i} + \frac{i}{\eta} \frac{\partial S}{\partial x_i} \right) \frac{\partial \Psi}{\partial x_i} \Big] d^n x \\ &= -\frac{\eta^2}{\mu^2} \int d^n x \, \Psi^* \frac{\partial}{\partial x_i} \left[\left(\frac{1}{2\rho} \frac{\partial \rho}{\partial x_i} + \frac{i}{\eta} \frac{\partial S}{\partial x_i} \right) \Psi \right] \\ &= -\frac{\eta^2}{\mu^2} \int d^n x \, \Psi^* \frac{\partial}{\partial x_i} \left\{ \frac{\partial}{\partial x_i} \left[\rho^{1/2} \exp\left(iS/\eta \right) \right] \right\} \\ &= -\frac{\eta^2}{\mu^2} \int d^n x \, \Psi^* \frac{\partial}{\partial x_i} \left(\frac{\partial}{\partial x_i} \Psi \right) \\ &= \int d^n x \, \Psi^* \left(-\frac{i\hbar}{m} \frac{\partial}{\partial x_i} \right)^2 \Psi \; .\end{aligned}$$